Summer Review Packet for AP CALCULUS

Directions: Complete the following problems. All work must be shown to receive full credit.

Simplifying by factoring

1. $2x^{-\frac{1}{2}} + 3x^{\frac{5}{2}}$

2. $3(x+1)^{\frac{1}{2}}(2x-3)^{\frac{5}{2}} + 7(x+1)^{\frac{3}{2}}(2x-3)^{\frac{3}{2}}$

3. $(x+2)^{1/2} + x(x+2)^{-1/2}$

4. $(2x-5)^{-3/4}(x+2) - (2x-5)^{1/4}$

Exponential and Logarithm Practice

1. Use the properties to find the exact value of the expression.

$$\log_5 8 + \log_5 50 - 4\log_5 2$$

Solve each equation. Use laws of logarithms.

2. $\log_2(\log_2 x) = 1$

3. $10^{2x} = 46$

4. $3e^{5x} = 18$

5. $\log(x+21) + \log x = 2$

Derivative Practice

Find the first derivative for each of the following.

1. $f(x) = \tan^3(4-2x)$

2. $\left(\frac{x+1}{2-x}\right)(2x-1)$

3. $y = \sin^3(5x^2)$

4. $f(x) = \frac{2x}{\sqrt{3+x^2}}$

5. $y = (2x^3+1)^2(x-5)^4$

6. $\frac{(3-x)(x^2-x-1)}{x^3}$

7. $(x^2+3)(x^3+4)$

8. $y = \left(\frac{2x}{1-x}\right)^4$

9. $y = x^2 \sin x$

10. $f(x) = -2\cos x + \sin^2 x$

Tangent Lines

1. Write an equation of the line tangent to the graph of $y = \cos(2x)$ at $x = \frac{\pi}{4}$.

2. Find $f(4)$ and $f'(4)$ if the tangent line to the graph of $f(x)$ at $x = 4$ has equation $y = 3x - 14$.

Calculate the second derivative.

1. $y = 12x^3 - 5x^2 + 3x$

2. $y = \sqrt{2x + 3}$

Compute $\frac{dy}{dx}$.

1. $x^3 - y^3 = 4$

2. $y = xy^2 + 2x^2$

Find all critical points of the function.

1. $f(x) = x^3 - \frac{9}{2}x^2 - 54x + 2$

Find the maximum and minimum values of the function on the given interval.

1. $y = 2x^2 - 4x + 2 \qquad [0, 3]$

Verify Rolle's Theorem for the given interval

1. $f(x) = x + x^{-1}, \qquad \left[\frac{1}{2}, 2\right]$

Find a point c satisfying the conclusion of the MVT for the given function and interval.

1. $y = \sqrt{x}, \qquad [4,9]$

Find the intervals of increase and decrease and relative extrema for the given function.

1. $y = x^3 - 6x^2$

Determine the intervals on which the function is concave up or down and find the points of inflection.

1. $y = x - 2\cos x$

2. $y = 4x^5 - 5x^4$

Related Rates:

1. Water pours into a conical tank of height 10ft and diameter of 8 ft at a rate of $10\ ft^3/\min$. How fast is the water level rising when it is 5 ft high?

Optimization

1. An open rectangular box with square base is to be made from 48 ft.^{2} of material. What dimensions will result in a box with the largest possible volume?

Graphing and Derivatives

1. Each graph in Figure 2 shows the graph of a function $f(x)$ and its derivative $f'(x)$. Determine which is the function and which is the derivative.

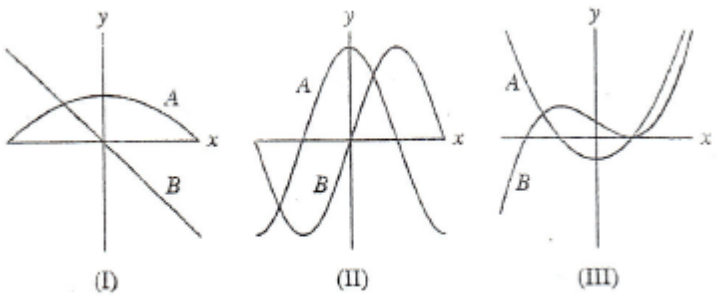

FIGURE 2 Graph of $f(x)$.

2. Figure 16 shows the graph of the derivative $f'(x)$ on $[0,1.2]$.
    a. Locate the points of inflection of $f(x)$ and the points where the relative maxima and minima occur.

    b. Determine the intervals on which $f(x)$ has the following properties:
        i. Increasing
        ii. Decreasing
        iii. Concave up
        iv. Concave Down

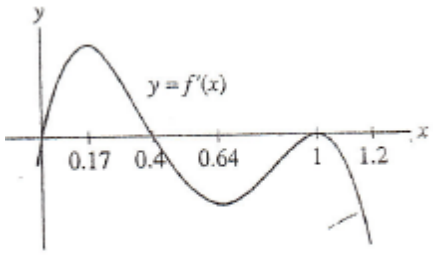

FIGURE 16

3. Match the description of $f(x)$ with the graph of its derivative $f'(x)$ in figure 1.
    a. $f(x)$ is increasing and concave up.
    b. $f(x)$ is decreasing and concave up.
    c. $f(x)$ is increasing and concave down.

(i) (ii) (iii)

FIGURE 1 Graphs of the derivative.