

Honors Algebra III/ Differential Calculus Summer Work (Review of Algebra II Concepts)

Directions: Use a separate sheet of paper. All work must be shown in order to receive full credit. If you worked with someone on these problems (friend, parent, sibling, tutor, etc), please indicate so on the top of your paper.

1. Write an equation for the line that passes through the point $(2, -5)$ and is perpendicular to the line $x + 3y = 4$.

Factor completely and simplify.

2. $\frac{1}{4}x^2 - x - 48$

3. $8x^{-\frac{1}{2}} + 3x^{\frac{1}{2}}$

4. $4x^2 + 28x + 49$

5. $8x^3 - 1$

6. $(2a + b)^2 - 100$

7. $x^3 + 3x^2 - 5x - 15$

Factor the greatest common factor from each polynomial.

8. $5(a - 1)^3 - 2(a - 1) + (a - 1)^2$

9. $(5w - 6)(w + 2) + (2w - 1)(w + 2)$

10. Write as a product of linear factors: $x^4 - 9$

Simplify.

11. $6(x + 5)^{-2} + (x + 5)$

12. $\frac{3+7i}{3-7i}$

13. $\sqrt[3]{54x^7y^{15}}$

14. $\frac{x-3}{\sqrt{x+1}+2}$

15. Factor and Simplify the expression: $\frac{3x(4)(2x-1)^4 - (2x-1)^6(3)}{(3x)^2(4x^2-4x+1)}$

16. Find the average of the two real numbers: $\frac{x}{2}$ and $\frac{x}{10}$

17. Solve the equation: $20x^3 - 500x = 0$

Find all the zeros (real and imaginary) of the function.

18. $f(x) = x^3 + x^2 - 4x - 4$

19. $f(x) = x^4 + 2x^3 + 3x^2 - 2x - 4$

Find the inverse of the function and write as a function of x .

20. $y = x^2 + 10x + 25$

21. $y = \frac{x-4}{x+3}$

22. Graph the function: $f(x) = -|x - 5| + 2$. Then give the domain and range of f .

23. Given $f(x) = x^2 - 2x - 3$, graph $y = |f(x)|$

Give the domain and range of each function.

24. $f(x) = x^2 - 6x + 10$

25. $g(x) = \sqrt{2x - 5}$

26. $h(x) = \frac{3}{x-2} + 4$

27. $k(x) = \sqrt[3]{x + 1} + 8$

28. Describe in words the transformations that map $f(x)$ onto $g(x)$.

$$f(x) = (x - 1)^2 + 2 \qquad g(x) = -(x + 3)^2 - 7$$

29. Given $f(x) = x^2 + 2x - 5$, find $f(k + 1) - f(k - 1)$.

Perform the indicated operation and simplify.

30. $\frac{2n^2 - 5n - 12}{n^2 - 10n + 24} \div \frac{4n^2 - 9}{n^2 - 9n + 18}$

31. $\frac{3}{a-2} - \frac{1}{2-a}$

32. $\frac{1}{b^2 - 5b + 6} + \frac{1}{b^2 - 4}$

33. $\frac{1 + \frac{1}{x}}{1 - \frac{1}{x}}$

Solve. Check for extraneous solutions.

34. $\sqrt{x + 3} = 1 + \sqrt{x + 1}$

35. $x + 2 = \sqrt{2x + 7}$

Summer Review Packet for AP CALCULUS

Directions: Complete the following problems. All work must be shown to receive full credit.

Simplifying by factoring

1. $2x^{-\frac{1}{2}} + 3x^{\frac{5}{2}}$

2. $3(x+1)^{\frac{1}{2}}(2x-3)^{\frac{5}{2}} + 7(x+1)^{\frac{3}{2}}(2x-3)^{\frac{3}{2}}$

3. $(x+2)^{\frac{1}{2}} + x(x+2)^{-\frac{1}{2}}$

4. $(2x-5)^{-\frac{3}{4}}(x+2) - (2x-5)^{\frac{1}{4}}$

Exponential and Logarithm Practice

1. Use the properties to find the exact value of the expression.

$$\log_5 8 + \log_5 50 - 4 \log_5 2$$

Solve each equation. Use laws of logarithms.

2. $\log_2(\log_2 x) = 1$

3. $10^{2x} = 46$

4. $3e^{5x} = 18$

5. $\log(x+21) + \log x = 2$

Derivative Practice

Find the first derivative for each of the following.

1. $f(x) = \tan^3(4 - 2x)$

2. $\left(\frac{x+1}{2-x}\right)(2x-1)$

3. $y = \sin^3(5x^2)$

4. $f(x) = \frac{2x}{\sqrt{3+x^2}}$

5. $y = (2x^3 + 1)^2(x - 5)^4$

6. $\frac{(3-x)(x^2 - x - 1)}{x^3}$

7. $(x^2 + 3)(x^3 + 4)$

8. $y = \left(\frac{2x}{1-x}\right)^4$

9. $y = x^2 \sin x$

10. $f(x) = -2 \cos x + \sin^2 x$

Tangent Lines

1. Write an equation of the line tangent to the graph of $y = \cos(2x)$ at $x = \frac{\pi}{4}$.

2. Find $f(4)$ and $f'(4)$ if the tangent line to the graph of $f(x)$ at $x = 4$ has equation $y = 3x - 14$.

Calculate the second derivative.

1. $y = 12x^3 - 5x^2 + 3x$

2. $y = \sqrt{2x + 3}$

Compute $\frac{dy}{dx}$.

1. $x^3 - y^3 = 4$

2. $y = xy^2 + 2x^2$

Find all critical points of the function.

1. $f(x) = x^3 - \frac{9}{2}x^2 - 54x + 2$

Find the absolute extrema of the function on the given interval.

1. $y = 2x^2 - 4x + 2$ $[0, 3]$

Verify Rolle's Theorem for the given interval

1. $f(x) = x + x^{-1}$, $\left[\frac{1}{2}, 2\right]$

Find a point c satisfying the conclusion of the MVT for the given function and interval.

1. $y = \sqrt{x}$, $[4, 9]$

Find the intervals of increase and decrease and relative extrema for the given function.

1. $y = x^3 - 6x^2$

Determine the intervals on which the function is concave up or down and find the points of inflection.

1. $y = x - 2\cos x$

2. $y = 4x^5 - 5x^4$

Related Rates:

1. Water pours into a conical tank of height 10ft and diameter of 8 ft at a rate of $10 \text{ ft}^3/\text{min}$. How fast is the water level rising when it is 5 ft high?

Optimization

1. An open rectangular box with square base is to be made from 48 ft^2 of material. What dimensions will result in a box with the largest possible volume?

Graphing and Derivatives

1. Each graph in Figure 2 shows the graph of a function $f(x)$ and its derivative $f'(x)$. Determine which is the function and which is the derivative.

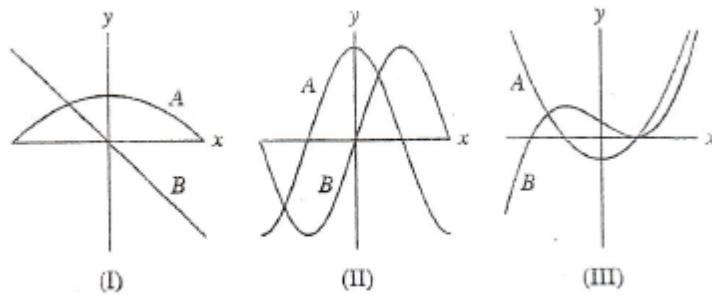


FIGURE 2 Graph of $f(x)$.

2. Figure 16 shows the graph of the derivative $f'(x)$ on $[0,1.2]$.
- Locate the points of inflection of $f(x)$ and the points where the relative maxima and minima occur.
 - Determine the intervals on which $f(x)$ has the following properties:
 - Increasing
 - Decreasing
 - Concave up
 - Concave Down

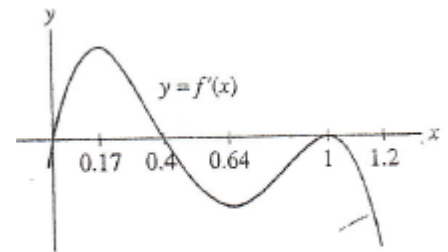


FIGURE 16

3. Match the description of $f(x)$ with the graph of its derivative $f'(x)$ in figure 1.
- $f(x)$ is increasing and concave up.
 - $f(x)$ is decreasing and concave up.
 - $f(x)$ is increasing and concave down.

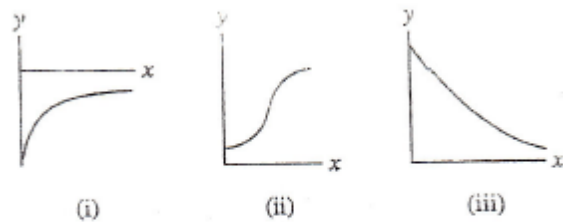


FIGURE 1 Graphs of the derivative.

AP Statistics Summer Work

Part 1: *The Joy of Stats* with Professor Hans Rosling

The following videos can be found on the website [gapminder.org](http://www.gapminder.org). The creator, Professor Hans Rosling, states on the site: "Gapminder is a non-profit foundation based in Stockholm. Our goal is to replace devastating myths with a fact-based worldview. Our method is to make data easy to understand..." Rosling uses innovative methods to present and display data and statistics.

Your assignment is to watch the video and **answer the questions below**.
(Feel free to watch more of the videos, most are short and interesting.)

<http://www.gapminder.org/videos/> - Scroll to the video that says "The Joy of Statistics"

- 1) What was the mean number of correct answers given by Swedish students to the questions:
'Which country has the highest child mortality rate?'
- 2) In displaying data on global health, what does the size of a country's bubble indicate?
- 3) What are two benefits of public statistics?
- 4) The word 'statistics' is derived from what word?
- 5) The first systematic collection of statistics is what document?
- 6) Along with average, when reporting data, what other value is important?
- 7) Which distribution models the number of buses that appear in a given hour?
- 8) Who used a polar area graph to display data?
- 9) What analytical method explores meaning and relationships within data?
- 10) What is a zettabyte?
- 11) Other than the internet, identify 3 technologies used to gather massive amounts of data.
- 12) How many bananas were eaten worldwide in the time it took to watch the video?

Part 2: An introduction to the first 3 chapters of the Statistics course

Use another sheet of paper for responses.

Chapter 1: Exploring Data with Displays

FOR ALL DISPLAYS, LABEL THE AXES AND TITLE THE GRAPH.

*Based on what you observe in a graph, describe the following:

***Overall pattern:** Give the following:

Center: the value that divides the data into 2 equally sized groups.
(half the values are smaller and half are larger)

Spread: Described as: "from (smallest value) to (largest value)."

Shape: key words to consider: symmetrical, skewed, single peaked, many peaked, etc.

Deviations: An individual observation that deviates from the overall pattern of the graph is an outlier (or extreme value). Use best judgement to determine whether outliers are present.

Display 1: STEMPLOT (or stem-and-leaf plot)

*Consider the following when making a stemplot:

-Each stem should have an equal number of possible leaves (equal intervals)

-Don't want too few stems (values are clustered) or too many stems (values are too spread out). 5 stems is a good minimum.

Example 1: The values below indicate the number of home runs hit by Babe Ruth, Hank Aaron, and Barry Bonds for the first 22 seasons. Construct a stemplot for the number of home runs hit by each player.

Ruth

0	4	3
2	11	29
54	59	35
41	46	25
47	60	54
46	49	46
41	34	22
6		

Aaron

13	27	26
44	30	39
40	34	45
44	24	32
44	39	29
44	38	47
34	40	20
12		

Bonds

16	25	24
19	33	25
34	46	37
33	42	40
37	34	49
73	46	45
45	5	26
28		

STEMPLOT

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2. Describe the **overall pattern** of each stemplot (Center, spread, shape) and deviations (outliers).

Ruth _____ **Aaron** _____ **Bonds** _____

3. Find the **mean** number of home runs hit in a year for each player.

Ruth: _____ **Aaron:** _____ **Bonds** _____

4. Eliminate the highest number of home runs for each player. Find the mean for each player for the remaining 21 seasons. **Compare** the change in mean among the three players.

Ruth _____ **Aaron** _____ **Bonds** _____

5. Find the **median** number of home runs for the first 22 seasons for each player.

Ruth: _____ **Aaron:** _____ **Bonds** _____

6. Eliminating the highest number of home runs for each player, find the median for each player for the remaining 21 seasons. **Compare** the change in median among the three players.

Ruth _____ **Aaron** _____ **Bonds** _____

7. Which player has the most **symmetric** distribution?

8. Which player seems to have the most **skewed** distribution?

9. For which player are the mean and median for all 22 seasons the closest?

10. For which player are the mean and median for all 22 seasons the farthest apart?

Display 2: HISTOGRAM

States differ widely with respect to the percentage of college students who are enrolled in public institutions. The U.S. Department of Education provided the accompanying data on this percentage for the 50 U.S. states for fall 1999.

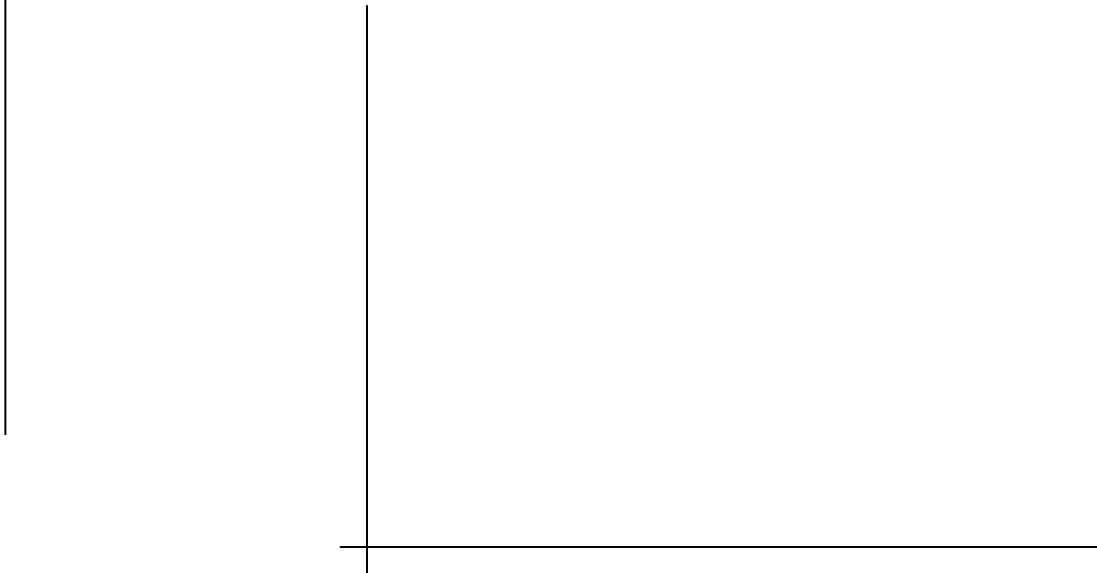
Percentage of College Students Enrolled in Public Institutions

95	81	85	72	73	74	79	84	89	63
91	86	89	92	87	90	83	84	89	96
87	85	76	84	80	95	75	81	73	82
81	77	75	70	55	56	87	88	82	81
84	76	80	56	55	43	52	62	80	82

1. Construct a *frequency table* (choose equally sized intervals such that you have at least 5 intervals)

Intervals | **Frequency**

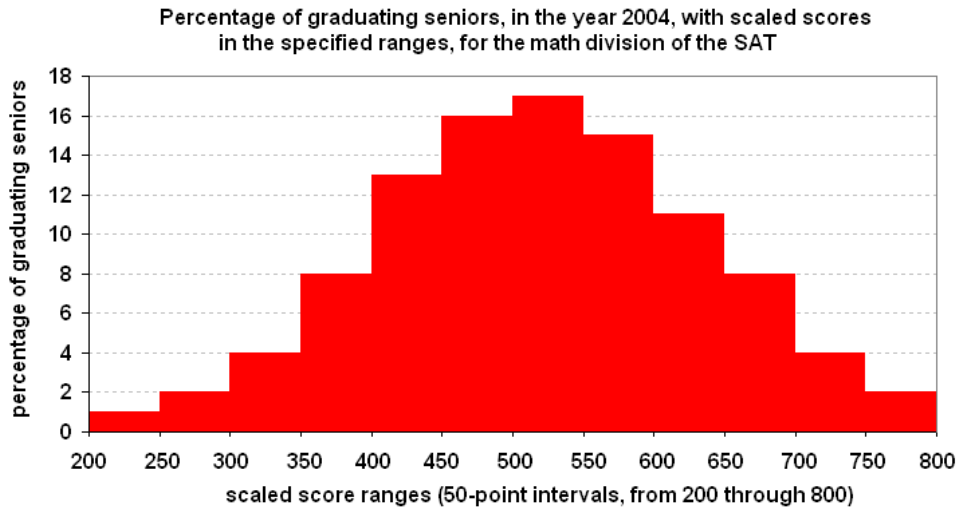
Display the information in a histogram.



3. Describe the distribution (Describe shape, center, spread and possible outliers)

Chapter 2: Introduction to Density Curves & the Normal Distribution

If the **overall pattern** in a graph (dotplot, stemplot, Histogram) is **regular**, a smooth curve can be used to describe the pattern.



- 1) A smooth curve can be drawn that gives the overall pattern of the distribution. Draw such a curve in the graph above.

Answer the following questions:

Example: What **proportion** of students got below a 400?

Total the frequencies of the bars in that range (200-400):

$$1\% + 2\% + 4\% + 8\% = 15\% \text{ of students got below a 400.}$$

- 2) What proportion of students scored:

A) between 400 and 600?

B) between 550 and 700?

C) 700 or higher?

Normal Curve

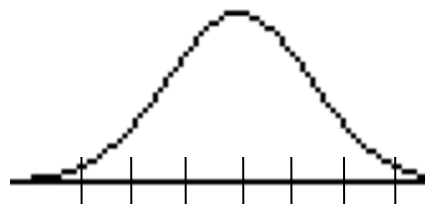
-A density curve that is **symmetric** and **bell shaped**.

-Is determined by the mean and standard deviation (S.D.)

Mean: determines the position of the curve. It is the location of the peak. Notation: μ (mu)

S.D.: determines the spread of the curve. There are approximately **3 standard deviations** in each direction from the mean. Notation: σ (sigma)

- 3) The distribution of scores of an IQ (intelligence quotient) test is normal. The mean score is 100 with standard deviation 15. **Label the normal curve** that represents IQ with values for the mean (μ) and 3 standard deviations in each direction from the mean.

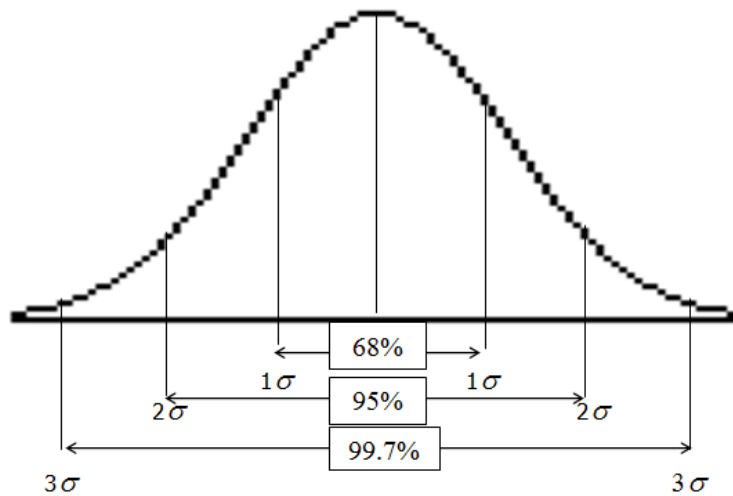


Empirical Rule (68 -95 – 99.7 Rule): All normal distributions obey a common rule:

68% of the observations fall within 1 standard deviation of the mean (in the interval $\mu \pm \sigma$)

95% of the observations fall within 2 standard deviations of the mean (in the interval $\mu \pm 2\sigma$)

99.7% of the observations fall within 3 standard deviations of the mean (in the interval $\mu \pm 3\sigma$)



4) A different IQ test has scores that are normally distributed with $\mu = 100$ and $\sigma = 16$. Label the normal curve above with the mean and standard deviation.

- a) What proportion of the population has scores between 84 and 116? _____
- b) What proportion has scores between 100 and 132? _____
- c) What proportion has scores between 68 and 84? _____
- d) What proportion has scores above 116? _____
- e) What proportion has scores below 52? _____

5) A normal distribution of Introduction to Biology Final exam scores at a large university has a mean of 77 and a standard deviation of 7.

a) Sketch and label a normal curve, showing the mean and 3 standard deviations in each direction.

b) Use the 68-95-99.7 rule to find the proportion of exam scores that are in the given interval:

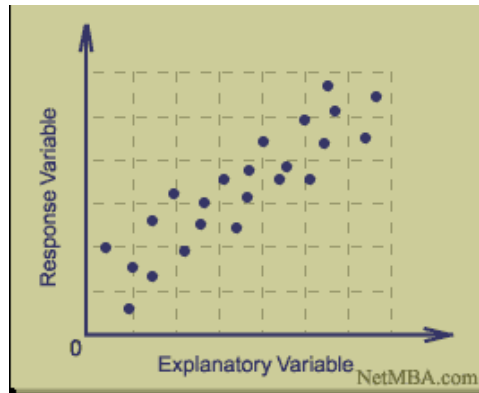
i) Between 70 and 84 ii) Between 63 and 77 iii) Between 70 and 91

iv) More than 84 v) Less than 63 vi) At least 91

Chapter 3: Comparing 2 variables (scatterplots)

-Show the relationship between two *quantitative variables* measured on the same individuals. Each individual in the data appears as a point in the plot.

-**Explanatory variable** on x-axis; **Response variable** on y-axis



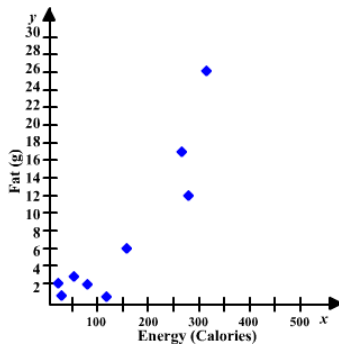
EXAMPLE

The table shows the amount of fat, amount of fiber, and the number of calories in a 4-oz serving of vegetables.

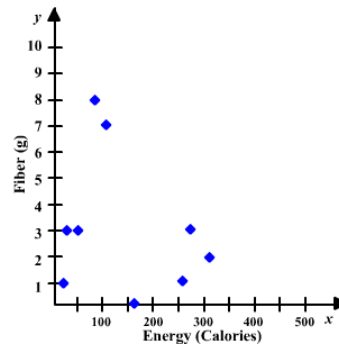
Nutritional Values for 4oz servings

Vegetable Item	Fat (grams)	Fiber (grams)	Energy (calories)
Martian Cabbage	3	3	52
Butter Casserole	26	2	328
Mauve Beans	2	8	82
Broccoli Tofu Casserole	12	3	276
Jalapeno Corn Husks	17	1	261
Rancid Eggplant Curry	1	7	123
Mixed Tuber Salad	2	1	21
Four-times-baked Potato	6	0	158
Steamed Mush	1	3	35

A) Comparing amount of **FAT** with the number of calories



B) Comparing amount of **FIBER** with the number of calories



- 1) What are the explanatory variable and response variable in graph (A)?
- 2) What are the explanatory variable and response variable in graph (B)?
- 3) Which graph seems to display a stronger relationship between the explanatory and response variables? Why?

To ***describe*** the relationship between an explanatory and a response variable, say:

- whether the association is linear or non-linear.
- whether the association is positive (increasing) or negative (decreasing).
- how strong the association is (use 1 or 2 words): weak, moderate, strong

- 4) Describe the relationship between the explanatory and response variables in graph (A).
- 5) Describe the relationship between the explanatory and response variables in graph (B).

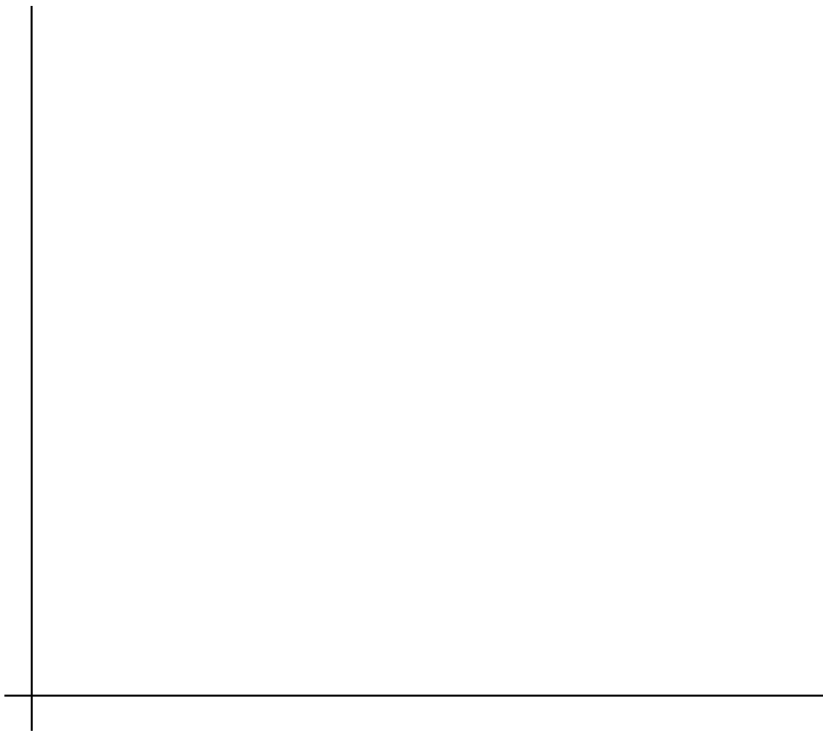
6) **SCATTERPLOT**

Construct a scatterplot showing the relationship among SAT Math and Verbal scores for the students in a Calculus class.

Let the math score be the explanatory variable (x-axis).

Let the verbal score be the response variable (y-axis).

Math score	Verbal score	Math score	Verbal score
660	520	740	670
590	600	790	570
800	590	710	580
680	600	670	600
660	570	690	580
620	670	690	570
650	590	800	570
600	560	710	560
660	660	740	630
760	610		



7) Describe the scatterplot. (i.e. describe the relationship between the two variables.)